## Exercise 3.2.4

Suppose that $f(x)$ is piecewise smooth. What value does the Fourier series of $f(x)$ converge to at the endpoint $x = -L$? at $x = L$?

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**Solution**

Assume that $f(x)$ is piecewise smooth on the interval $-L \leq x \leq L$. If $f(-L) = f(L)$, then the Fourier series converges to $f(-L) = f(L)$ at the endpoints. If $f(-L) \neq f(L)$, then the Fourier series converges to the average of these two values at $x = \pm L$:

$$\frac{f(-L) + f(L)}{2}.$$